# EXPONENTIAL GROWTH AND DECAY

SECTIONS 7.1 & 7.2

Algebra 2

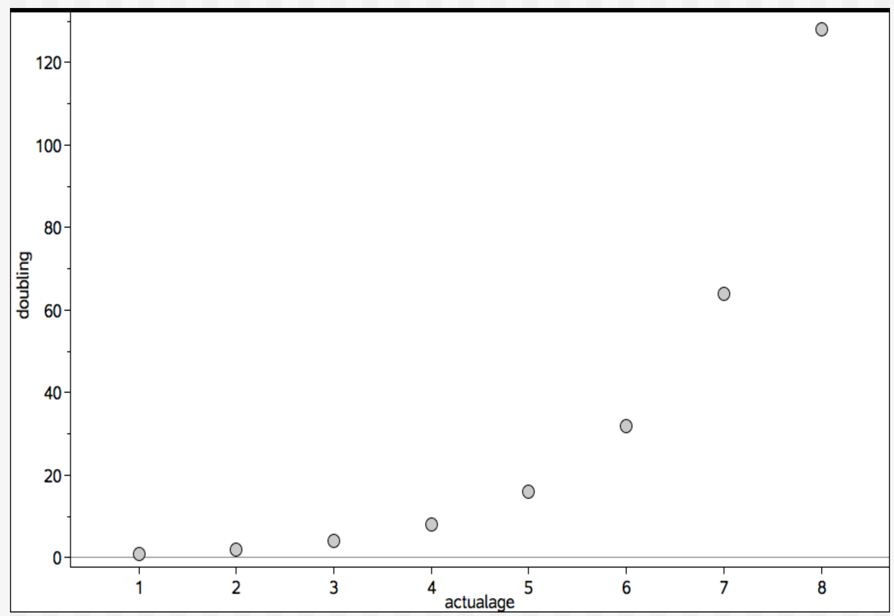
Mr. Keltner

# Funny, but true?

- “When I turned two I was really anxious, because I'd doubled my age in a year. I thought, if this keeps up, by the time I'm six I'll be ninety.” -- Steven Wright
  - The same man who said “So how can you tell when you’re out of invisible ink?”
- Clever, but how accurate is his reasoning?

# Exponential Growth

- The graph at the right shows Steven's **actual** age as the x-values and his **proposed** age as the y-values
- The table below it shows the values that go along with his actual age
- So, was he actually 90 when he was six?



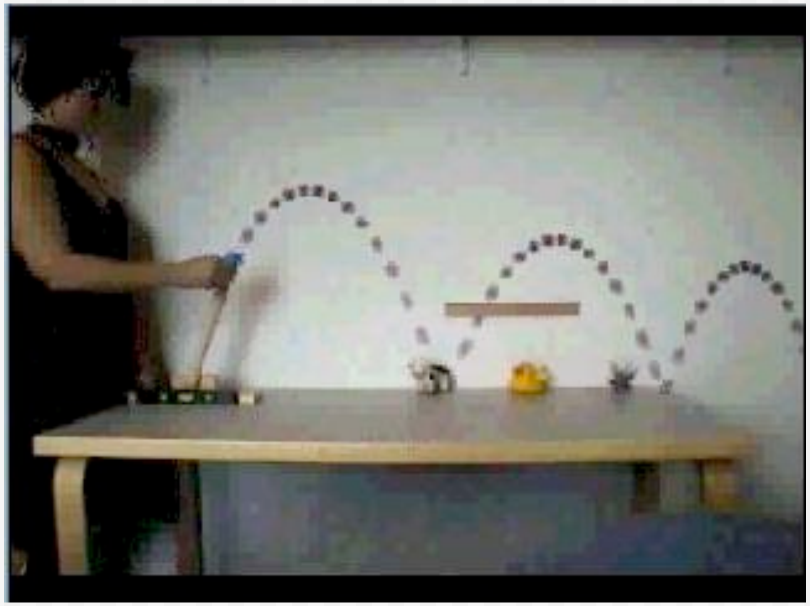

| ***Actual Age (yrs.)*** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| ***"Doubling" age*** | 1 | 2 | 4 | 8 | 16 | 32 | 64 | 128 |

# Exponential Growth

- Assuming an initial (starting) age of 1 year old, we wanted to predict his "age" after 6 years.
- His "age" ($y$-value) after $x$ actual years could be represented by the exponential expression:
  - $1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 = 1 \cdot 2^{x-1}$
  - The expression ,$1 \cdot 2^{x-1}$, is called an ***exponential expression***, because the exponent, $x$, is a variable and the base, 2, is a fixed number (a constant).
  - The ***base*** of an exponential expression is commonly referred to as a ***multiplier.***
    - Section 7.1 refers to it as a ***growth factor***, since it shows a growth in values.

# What if there is no growth?

- If there is no pattern of growth (or increase in values), what do we do different?
- An example of this would be a racquetball bouncing from a starting height of 3 feet.
  - Each time the ball bounces, it rebounds to $^{2}/_{3}$ the height it did before.
  - Ideally, the ball would continue bouncing in this pattern infinitely, but would be so close to the ground we could not measure its height.



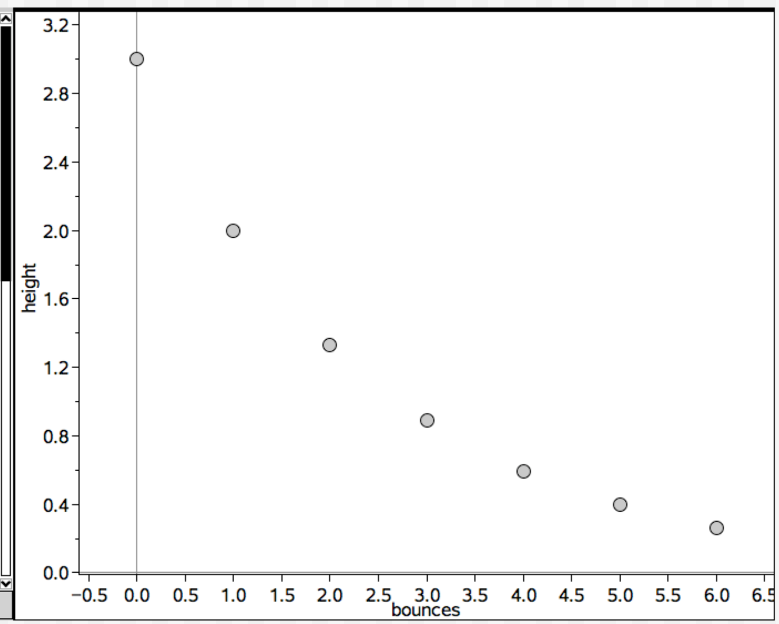
- This is an example of **exponential decay.**
  - ***Decay*** because it shows a decrease in values.
  - Each value is a certain *factor* of the value before it.

# Racquetball example

- If the initial height is 3 feet and it rebounds to $^2/_3$ that height after 1 bounce, it will bounce to 2 feet high.
- The ball would bounce $^2/_3$ that height for its second bounce, or $1^1/_3$ feet high.
- The graph and table for this example are at the right.
- Ideally, this pattern would continue indefinitely.
  - In a real world, the ball stops bouncing after a certain amount of time.

| | A bounces | B height | C |
|---|---|---|---|
| ♦ | =seq(x,x,( | =3*(2/3)^b | |
| 1 | 0 | 3 | |
| 2 | 1 | 2 | |
| 3 | 2 | 4/3 | |
| 4 | 3 | 8/9 | |
| 5 | 4 | 16/27 | |
| 6 | 5 | 32/81 | |
| 7 | 6 | 64/243 | |

B1 =3



# Exponential Growth & Decay

- In general, we can set up a formula for ***exponential growth and decay***

$$y = y_0 \bullet m^x$$

  - $y_0$ (Said "y-sub not") is the initial or beginning y-value (when x = 0)
  - ***m*** is the multiplier (the base) or the factor by which each value increases or decreases
  - Note that our variable, ***x***, appears as part of the exponent of our multiplier.

# Exponential Growth & Decay

- How do we know which is growth and which decay?
  - Consider 3 scenarios:
  - If ***m* = 1**?
    - **1**$^x$ = 1 for any value of x.
    - This would be a ***constant function*** where $y = y_o \cdot 1^x = y_o$
  - If ***m* < 1**?
    - We are multiplying by something *smaller* than 1, so the end value will be smaller.
    - This is ***exponential decay***.
  - If ***m* > 1**?
    - We are multiplying by something *bigger* than 1, so the end value will be bigger.
    - This is ***exponential growth***.

# Identify the multiplier

- If you are given the rate at which a function is increasing or decreasing, find the multiplier for the function.
- Remember, a constant function will multiply by 1, which is the same as 100%.

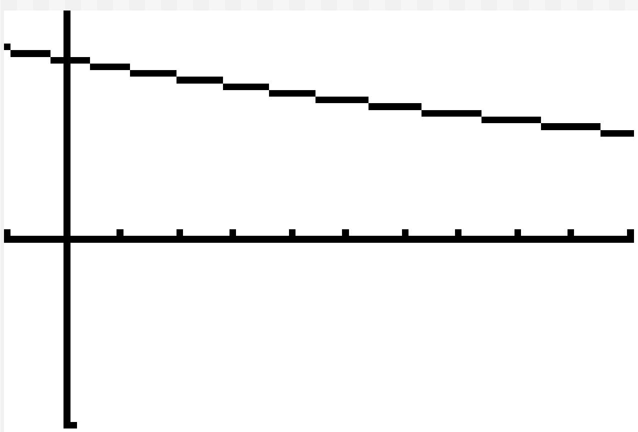
- Examples
- 5% growth
  - 5% is the factor of growth, and 100% + 5% = 105%.
  - The multiplier would be 1.05.
- 12.75% decay
  - 12.75% is the factor of decay, and 100% - 12.75% = 87.25%.
  - The multiplier would be .8725.

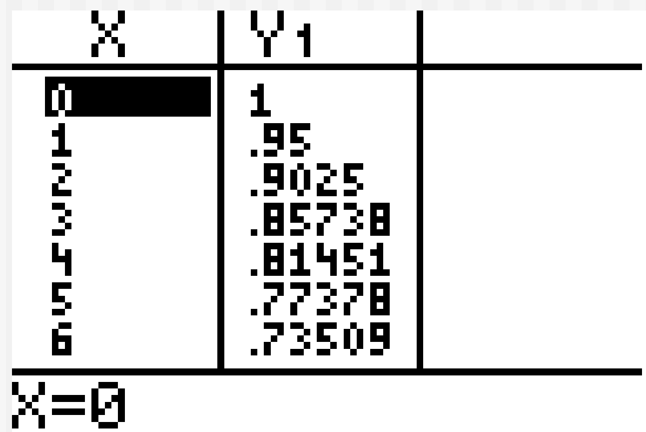
# Rechargeable Battery Example

- Rechargeable batteries are great, but can't last forever. Suppose the power capacity for Rayovac batteries decreases 5% with each time they are recharged.
  - Let x represent the number of recharges and y represent the power capacity in relation to the original capacity (as a decimal).
  - Write an equation to model the power capacity of the rechargeable battery.
  - Find the percent of power that will be held after 5 recharges.
  - Find the percent of power after 7 recharges.

# Rechargeable Battery Solution

- Since the original power capacity is 100%, let $y_0 = 1.0$.
- The capacity decreases 5% each charge, so let $m = .95$.
- Our equation is $y=1\cdot(.95)^x$
- After 5 recharges, $y=1\cdot(.95)^5 \approx .774$
- After 7 recharges, $y=1\cdot(.95)^7 \approx .698$



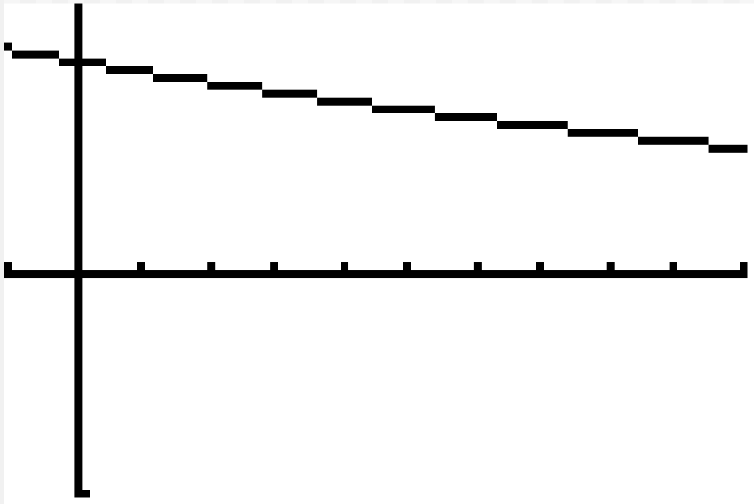


| X | Y1 |
|---|---|
| 0 | 1 |
| 1 | .95 |
| 2 | .9025 |
| 3 | .85738 |
| 4 | .81451 |
| 5 | .77378 |
| 6 | .73509 |

X=0

# Rechargeable Battery Solution

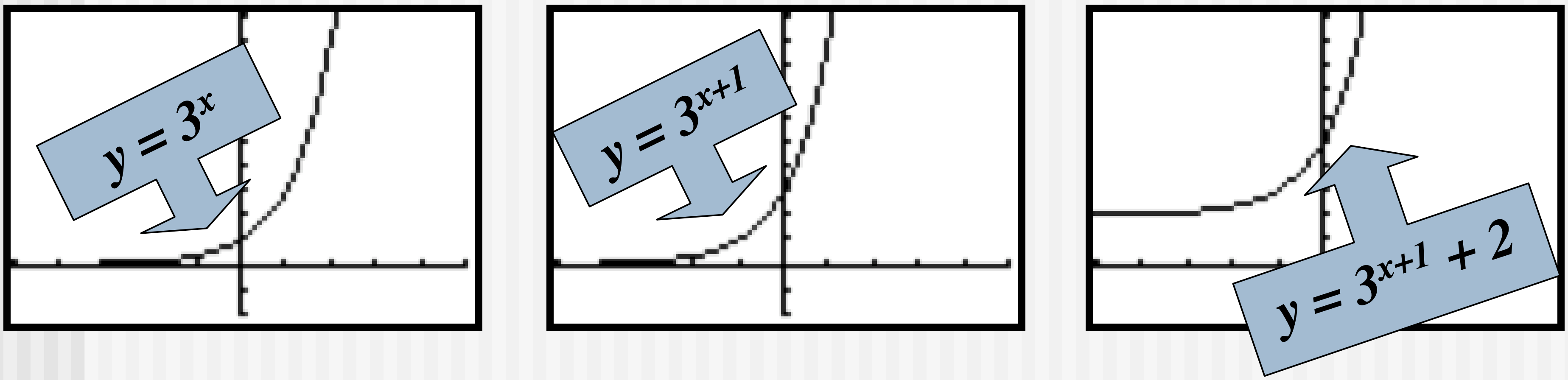
- Why does the graph of the equation $y = 1 \cdot (.95)^x$ look like it is a straight line?
  - Simply put, it is because the multiplier, 0.95, is so close to 1, that there is not a significant amount of decrease from one *y*-value to the next.



# Bacteria Example

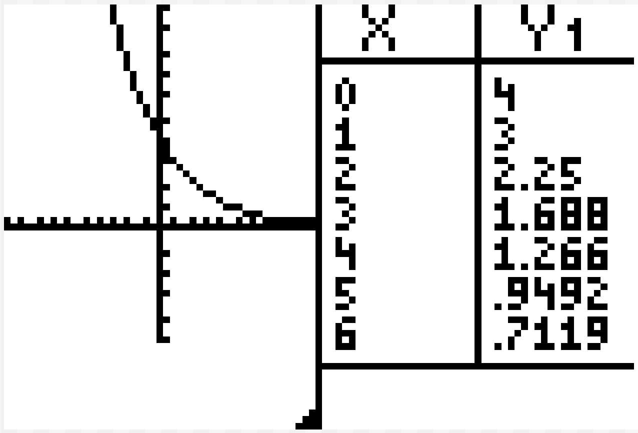
- Suppose 225 bacteria triple in amount every hour.
- Predict the population of bacteria for the situation for each time period.
  - After 3 hours
    - $y=225 \cdot (3)^x$
    - $x = 3$, so $y=225 \cdot (3)^3=225 \cdot 27=$ ***6075 bacteria***
  - After 4.5 hours
    - A decimal exponent?  Can we do that?
    - Remember your “roots and branches” tip to simplify rational exponents.
    - Could be evaluated on a calculator easily.
    - $x = 4.5$, so $y=225 \cdot (3)^{4.5}=225 \cdot 140.296 \approx$ ***31567 bacteria***

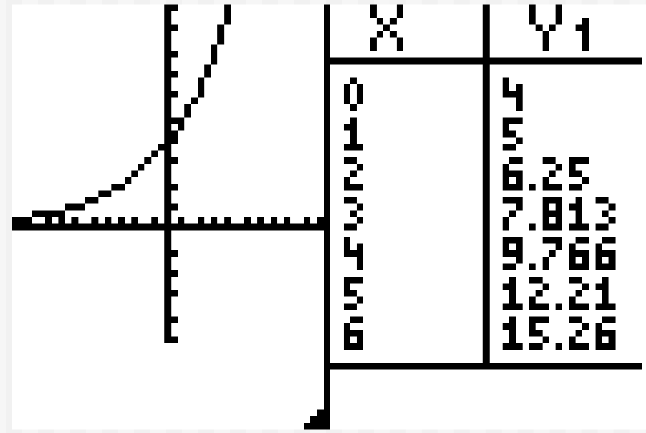
# Translations of Exponential Functions




- To graph a function of the form $y = a \bullet b^{x-h} + k$:
  - Graph its parent function $y = a \bullet b^{x}$.
  - Then shift the graph horizontally ***h*** units.
    - "Add to the *x*, move to the west."
  - Also shift the graph vertically ***k*** units.
    - "Add, like they *y*, move to the sky."

# Recap: Decay vs. Growth




- ***Decay***
  - Base less than 1
  - Horizontal asymptote at $y = 0$
  - Fits the equation $\boldsymbol{y=y_0 \cdot m^x}$ where $0 < m < 1$
- ***Growth***
  - Base greater than 1
  - Horizontal asymptote at $y = 0$
  - Fits the equation $\boldsymbol{y=y_0 \cdot m^x}$ where $m > 1$

# ASSESSMENT

Worksheets 7.1 & 7.2,

**Evens only**