

Math Resource Center
COLLEGE ALGEBRA FINAL REVIEW WORKSHEET
October 2009

1. Identify the center and radius of this circle. Sketch the graph of the circle. $(x - 3)^2 + (y + 4)^2 = 4$

2. Sketch the graph of each function. State the domain and range.

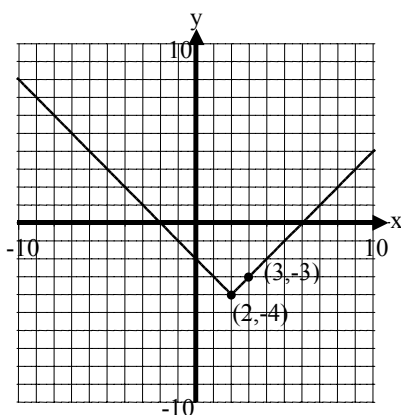
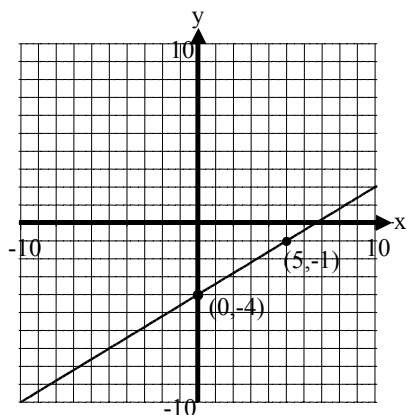
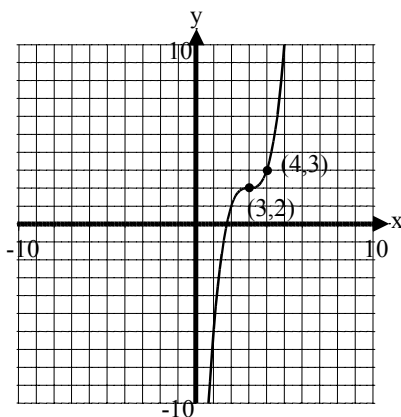
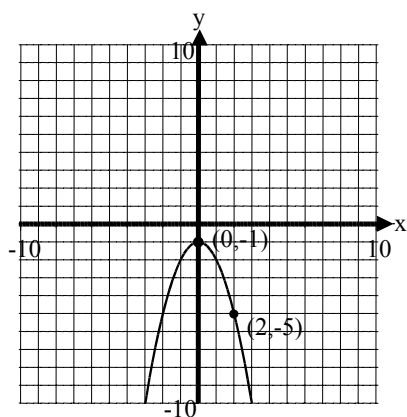
a. $y = -2(x - 1)^2 + 5$

c. $y = -\sqrt{-x + 2} - 1$

b. $y = -|x - 2| - 3$

d. $y = \frac{1}{2}(x - 3)^3 + 1$

3. Write the function represented by each graph.



4. Given that $f(x) = 4x + 5$

a. find $f(-2)$

b. find $f(3x + 1)$

5. Given that $f(x) = x - 2$ and $g(x) = 2x^2 - 5x - 3$

a. find $f \circ g$

c. find $g(f(1))$

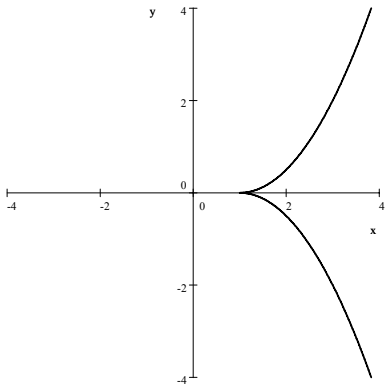
b. find $(f + g)(x)$

d. find $(f \cdot g)(3)$

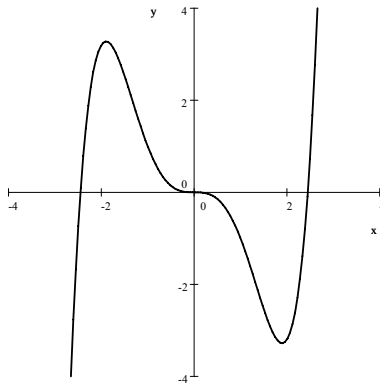
6. For each of the following graphs answer these questions.

- I. Does the graph show any symmetry? If so, what type?
- II. Does the graph represent y as a function of x ?
- III. If so, does the function have an inverse?

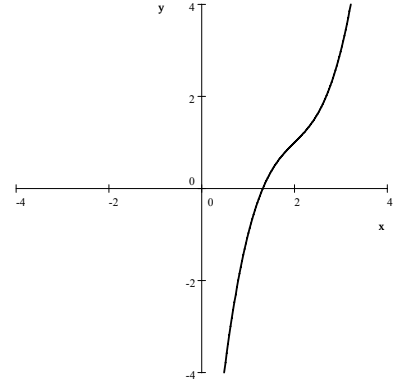
a.



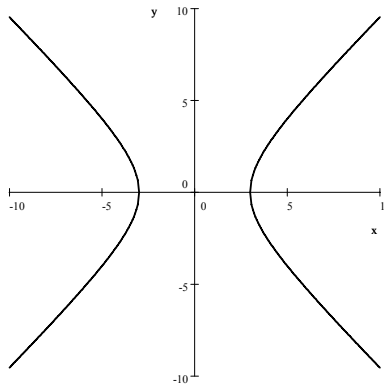
b.



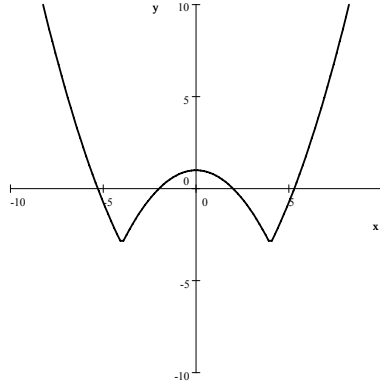
c.



d.



e.



7. Find $f^{-1}(x)$, for each of these functions.

a. $f(x) = \sqrt[3]{x-2}$

b. $f(x) = \frac{9}{5}x + 27$

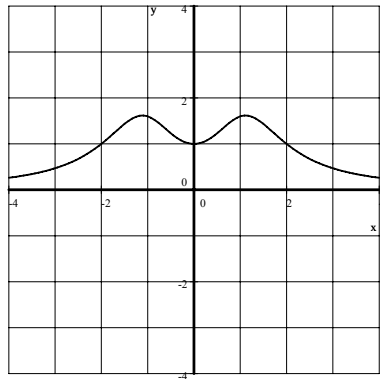
8. Given the graph of $f(x)$, sketch the graph of

a. $g(x) = f(x) + 2$

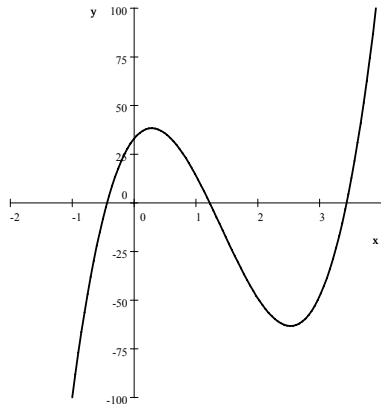
c. $h(x) = f(x - 1)$

b. $k(x) = -f(x)$

d. $m(x) = -f(x + 2) - 1$



9. Perform these tasks by referring to the function $f(x) = -2x^2 + 6x + 11$.
- find the vertex of the parabola
 - find the y -intercept
 - find the x -intercept(s)
 - sketch the graph of the parabola
10. A pool company treats pools chemically for algae. If the algae content is given by $A(t) = 20t^2 - 800t + 8500$, where t is the number of days after the pool has been treated, how many days after a treatment will the algae content be a minimum?
11. A field that borders a river is to be fenced. A rectangular enclosure will be set up with one of its sides being the river. If 500 meters of fencing is used for the three sides that need fence [don't put a fence along the river], what is the maximum area that can be enclosed?
12. Find a polynomial function with integer coefficients that has the following zeros.
- $x = 3$ $x = \frac{3}{4}$ $x = -5$ c. $x = -2 + 5i$ $x = 1$
 - $x = 4 \pm \sqrt{3}$ $x = 0$
13. Given the function $f(x) = 18x^3 - 76x^2 + 39x + 33$ and its graph,
- list all possible rational zeros.
 - use synthetic division to verify that there is a rational zero between 1 and 2.



14. Solve the equation $12x^4 + 41x^3 - 108x^2 - 59x + 30 = 0$.
15. Find all real and complex zeros of the function $f(x) = x^4 - 4x^2 - 32$.
16. Find all real and complex zeros of the function $f(x) = x^4 - 10x^3 + 34x^2 - 42x + 9$.
17. Find all real and complex zeros of the function $f(x) = 2x^3 + 13x^2 + 56x + 25$.
18. Sketch each of these functions. Identify the domain and all intercepts and asymptotes.
- $f(x) = \frac{6}{x - 4}$
 - $f(x) = \frac{x - 1}{x^2 - 4}$
 - $f(x) = \frac{4x + 5}{x + 3}$
 - $f(x) = \frac{x^2 + 5x + 4}{x - 1}$

19. Sketch each of these functions. Identify the domain and range of each.

a. $f(x) = 2^{-x+3} - 5$

c. $f(x) = -\log_6(x + 1) - 2$

b. $f(x) = e^{x+2} + 1$

d. $f(x) = \ln(x - 2) + 4$

In problems #20 - 28, include a decimal approximation rounded to three decimal places with any irrational solutions.

20. Solve the equation $\log x + \log(2x + 1) = 1$

25. Solve the equation $3^{4x} = 75$

21. Solve the equation $\log_4(x - 2) - \log_4(x - 3) = 2$

26. Solve the equation $8(4^{x-5}) = 60$

22. Solve the equation $\log_4(3x + 5) = 3$

27. Solve the equation $5 + 3e^x = 41$

23. Solve the equation $\ln x - \ln(x - 3) = \ln 3$

28. Solve the equation $1000e^{-0.08t} = 5000$

24. Solve the equation $8^{3x-2} = 64$

29. An investment of \$5,000 averages an interest rate of 8.5% when invested in a stock based mutual fund. Determine the doubling time for this investment if the interest is compounded continuously.

30. An investment of \$1,500 averages an interest rate of 4.2% when invested in a government savings bonds. If the interest is compounded continuously, how long will it take for the investment to grow to \$4000?

31. An investment of \$15,000 averages an interest rate of 10.5% when invested for 7 years in a growth mutual fund. Determine the final balance in each of the following situations.

a. The interest is compounded quarterly.

b. The interest is compounded continuously.

32. The half-life of the radioisotope, I^{131} , used in the treatment of hyperthyroidism, is 8.1 days. How much of a 100 mg sample is still radioactive after 5 days?

33. The population of a city is given by $P = 152,000e^{0.012t}$ where $t = 0$ corresponds to the present day.

a. According to this model, what will the population be in 20 years?

b. How long will it take for the population to reach 200,000. Round your answer to the nearest tenth of a year.

34. Solve this system of equations.

$$y = 3x^2 - 2x - 1$$

$$2x + 3y = 2$$

36. Solve this system of equations.

$$2x + 2y - 6z = -5$$

$$2x - y + 4z = 11$$

$$4x + y - 2z = 1$$

35. Solve this system of equations.

$$2x + 3y - z = -5$$

$$3x - y + z = 0$$

$$x + 4y + 2z = 5$$

37. Solve this system of equations.

$$3x + 2y - z = 2$$

$$9x - 4y + 3z = 3$$

$$-3x + 6y - 2z = 4$$

38. Solve this system of equations.

$$x - y + 3z = 1$$

$$3x + 2y - z = 8$$

$$6x + 5y - 4z = 17$$

39. Graph this system of linear inequalities.

$$2x - 3y \leq 36$$

$$-2 \leq x \leq 5$$

$$11x + 5y \leq 55$$

40. Write the first five terms of the sequences defined by these formulas.

a. $a_1 = 4$ $a_{n+1} = 5a_n - 2$

c. $a_n = 3\left(\frac{1}{2}\right)^{n-1}$

b. $a_n = \frac{n}{(n-1)!}$

41. Given that each of the following sequences are either arithmetic or geometric, find the indicated term.

a. 25th term of {3, 7, 11, 15, 19, ...}

b. 20th term of {270, 90, 30, 10, ...}

42. Find the sum of these infinite geometric series.

a. $16 + 8 + 4 + 2 + 1 + \dots$

b. $135 - 45 + 15 - 5 + \dots$

43. Find the indicated sums.

a. $\sum_{k=2}^6 (2k^2 - k)$

c. $\sum_{k=1}^{50} (-3k - 5)$

b. $\sum_{k=1}^{12} 5\left(\frac{4}{3}\right)^{k-1}$

d. $\sum_{k=0}^{\infty} \left(-\frac{2}{5}\right)^k$

44. Expand $(2x - 3y)^6$ using the Binomial Theorem or Pascal's Triangle.

45. Expand $(5a + 4b)^4$ using the Binomial Theorem or Pascal's Triangle.

46. Find the coefficient a of the term ax^5 in the expansion of $(5x + 2)^{12}$.

These formulas will be provided to you on your final!

Arithmetic: $a_n = a_1 + (n - 1)d$ or $a_n = dn + c$

$$S_n = \frac{n}{2}(a_1 + a_n) \text{ or } \frac{n}{2}(2a_1 + (n - 1)d)$$

Geometric: $a_n = a_1 r^{n-1}$

$$S_n = a_1 \frac{(1 - r^n)}{1 - r}$$

$$S = \sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1 - r} \quad |r| < 1$$

10. 20 days

11. 31,250 m²

12. a. $f(x) = (x - 3)(x + 5)(4x - 3)$
 or $f(x) = 4x^3 + 5x^2 - 66x + 45$

b. $f(x) = x(x^2 - 8x + 13)$
 or $f(x) = x^3 - 8x^2 + 13x$

c. $f(x) = (x - 1)(x^2 + 4x + 29)$
 or $f(x) = x^3 + 3x^2 + 25x - 29$

13. a. $\pm \left(1, 3, 11, 33, \frac{1}{2}, \frac{3}{2}, \frac{11}{2}, \frac{33}{2}, \frac{1}{3}, \frac{11}{3}, \frac{1}{6}, \frac{11}{6}, \frac{1}{9}, \frac{11}{9}, \frac{1}{18}, \frac{11}{18} \right)$

b. By synthetic division, $\frac{11}{9}$ is verified as the rational zero between 1 and 2.

14. $x = 2 \quad x = -5 \quad x = \frac{1}{3} \quad x = -\frac{3}{4}$

15. $x = \pm 2\sqrt{2} \quad x = \pm 2i$

16. $x = 3 \quad x = 2 \pm \sqrt{3}$

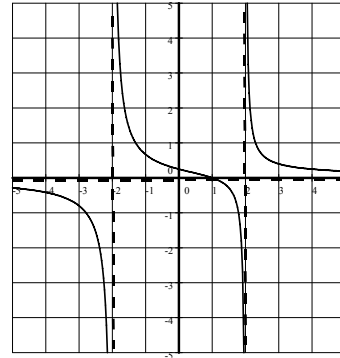
17. $x = -\frac{1}{2} \quad x = -3 \pm 4i$

18. b.

domain $x \neq \pm 2$

intercepts $\left(0, \frac{1}{4} \right) (1, 0)$

asymptotes $x = \pm 2 \quad y = 0$

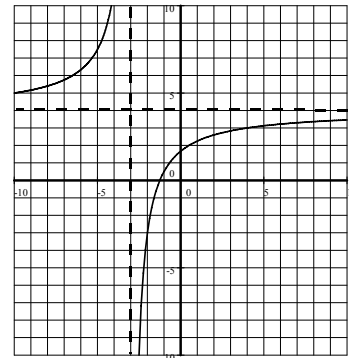


18. c.

domain $x \neq -3$

intercepts $\left(0, \frac{5}{3} \right) \left(-\frac{5}{4}, 0 \right)$

asymptotes $x = -3 \quad y = 4$

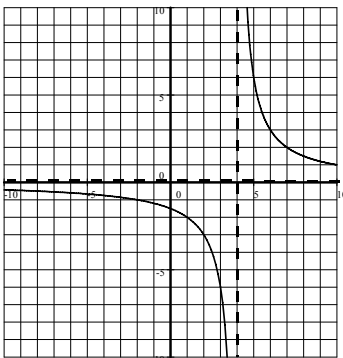


18. a.

domain $x \neq 4$

intercepts $\left(0, -\frac{3}{2} \right)$

asymptotes $x = 4 \quad y = 0$

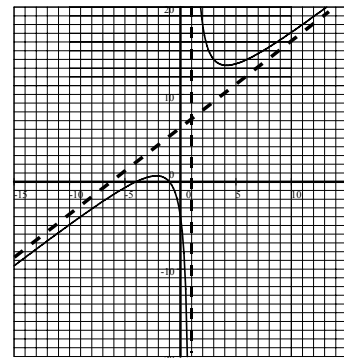


18. d.

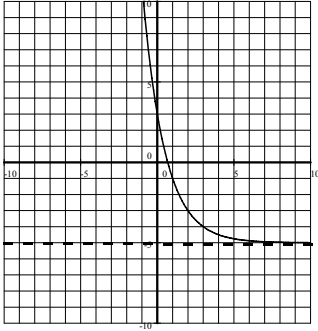
domain $x \neq 1$

intercepts $(0, -4) (-1, 0) (-4, 0)$

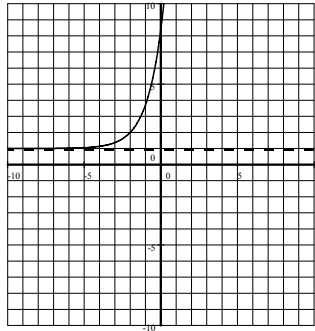
asymptotes $x = 1 \quad y = x + 6$



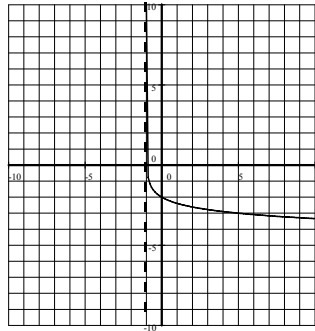
19. a. domain $(-\infty, \infty)$
range $(-5, \infty)$



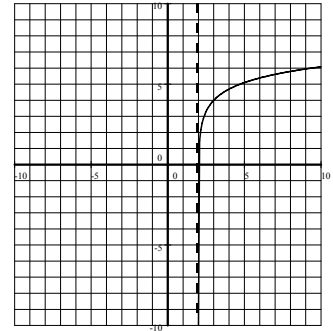
19. b. domain $(-\infty, \infty)$
range $(1, \infty)$



19. c. domain $(-1, \infty)$
range $(-\infty, \infty)$



19. d. domain $(2, \infty)$
range $(-\infty, \infty)$



20. $x = 2$

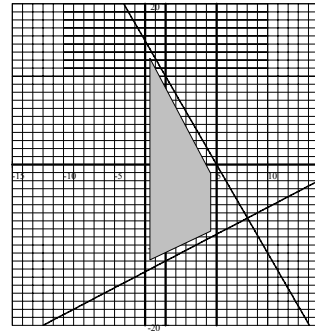
21. $x = \frac{46}{15}$

22. $x = \frac{59}{3}$

23. $x = \frac{9}{2}$

24. $x = \frac{4}{3}$

39.



25. $x = \frac{\ln 75}{4 \ln 3} \approx 0.982$

26. $x = \frac{\ln 7.5}{\ln 4} + 5 \approx 6.453$

27. $x = \ln 12 \approx 2.485$

28. $t = \frac{\ln 5}{.08} \approx 20.118$

40. a. 4 18 88 438 2188

b. 1 2 $\frac{3}{2}$ $\frac{2}{3}$ $\frac{5}{24}$

c. 3 $\frac{3}{2}$ $\frac{3}{4}$ $\frac{3}{8}$ $\frac{3}{16}$

41. a. 99 b. $\frac{10}{43,046,721} \approx 2.323 \times 10^{-7}$

29. 8.15 years

34. $(1, 0) \left(-\frac{5}{9}, \frac{28}{27} \right)$

42. a. 32

b. $\frac{405}{4} = 101.25$

30. 23.35 years

35. $(-1, 0, 3)$

43. a. 160

c. -4,075

31. a. \$30,987.04
b. \$31,282.23

36. no solution

b. ≈ 458.539

d. $\frac{5}{7}$

32. 65.2 mg

37. $\left(\frac{1}{3}, \frac{3}{2}, 2 \right)$

33. a. 193,229 people
b. 22.9 years

38. $(2 - a, 2a + 1, a)$

44. $64x^6 - 576x^5y + 2160x^4y^2 - 4320x^3y^3 + 4860x^2y^4 - 2916xy^5 + 729y^6$

45. $625a^4 + 2000a^3b + 2400a^2b^2 + 1280ab^3 + 256b^4$

46. 316,800,000