Hypothesis Tests for One Population Proportion

Section 12.2
Statistics, MATH 181
Mr. Keltner

Recap of This Unit

- The goal of section 12.1 was to accurately obtain a confidence interval for a population proportion, $\boldsymbol{p}$, by only taking a sample of the population, represented by a sample proportion, $\hat{\boldsymbol{p}}$.
- This section discusses performing hypothesis tests for a population proportion.
  - This process is just a special case of the one-mean $\boldsymbol{z}$-test.
  - We start by defining a formula for a standardized version of $\hat{\boldsymbol{p}}$ *(a z-score in terms of the value of the sample proportion, $\hat{\mathbf{p}}$).*

Standardized value of $\hat{p}$

- From our properties of sample proportions in section 12.1, we can say that:
  - For a large ***n***, (when ***n•p*** and ***n•(1-p)*** are each 5 or greater), the ***standardized version of*** $\hat{p}$ is:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

  - Also, we assume that this gives us enough evidence to say that this standardized version of $\hat{p}$ has approximately the standard normal distribution (with $\mu = 0$ and $\sigma = 1$).
  - This helps establish the test statistic to compare with the critical value(s) in our hypothesis test.

One-Proportion *z*-Tests

- To perform a large-sample hypothesis test where the null hypothesis is $\boldsymbol{H_0: p = p_0}$, we use the variable

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

  as the test statistic and obtain the critical value(s) from the standard normal table, Table II (inside back cover of textbook).
- This procedure is called the ***one-proportion z-test***.

Example 1

- From the information given below, determine whether using the one-proportion *z*-test is appropriate, or if it is not a reasonable method of estimating the population proportion.
  - $x = 25$
  - $n = 80$
  - $H_0: p_0 = 0.5;$
  - $H_a: p_0 \neq 0.5.$

Example 2

- A hypothesis test is to be performed for a population proportion. From the given information at right, calculate the value of the test statistic,

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

- Out of 93 observations, 65% were successes.
- H_0: $p_0 = 0.6$

Steps for a One-Proportion *z*-Test

- We must be sure that:
  - Our sample comes from a simple random sample
  - Both $n \cdot p_0$ and $n \cdot (1-p_0)$ are at least 5
- The null hypothesis is $H_0: p = p_0$, and we must establish an alternative hypothesis.
- Determine the significance level, α.
  - (Remember the confidence level will be 1-α.)
- Compute the value of the test statistic, represented by z_0.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

- If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.
- Interpret your results.

Example 3

- Use the one-proportion *z*-test to perform the specified hypothesis test.
  - $x = 17$
  - $n = 100$
  - H_0: $p = 0.25$
  - H_a: $p \neq 0.25$
  - $\alpha = 0.01$

$$\hat{p} = 0.17$$

$$z_{\alpha/2} = 2.575$$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.17 - 0.25}{\sqrt{\frac{0.25(1-0.25)}{100}}} = -1.848$$

Since our value of the test statistic is within the confidence level specified, we ***do not reject H_0***.

Assessment

Pgs. 627-628:

#'s 12.51 - 12.58