The Multiplication Rule; Independence

Section 4.6

Statistics, MATH 181

Mr. Keltner

Finding A Way

- From the ***conditional probability rule*** in Section 4.5, $$P(B|A) = \frac{P(A \& B)}{P(A)}$$
- We could compute conditional probabilities in terms of unconditional probabilities.
- By simply multiplying by the denominator, we create another formula that we focus on in this section, the ***general multiplication rule***: $P(A \& B) = P(A) \bullet P(B|A)$

General Multiplication Rule, Explained

- For any two events *A* and *B*, the probability that both occur, ***P*(*A* & *B*)**, equals the probability that one event occurs, ***P*(*A*)**, times the conditional probability of the other event, given the specified event, ***P*(*B*|*A*)**.

$$P(A \,\&\, B) = P(A) \bullet P(B|A)$$

  - In some instances, it is easier to use marginal probabilities and conditional probabilities in order to obtain a joint probability.

Example 1(Pg. 202, #4.109)

- According to the Opinion Research Corporation, 44% of U.S. women suffer from holiday depression and, from the U.S. Census Bureau's ***Current Population Reports***, 52% of U.S. adults are women.
- Find the probability that a randomly selected U.S. adult is a woman who suffers from holiday depression.
  - Express your answer in terms of a percentage.

Tree diagrams

- A ***tree diagram*** can be helpful in applying the general multiplication rule.
  - It can help organize our data to show the probability of successive events (one trial after another), as well as see the impact of one event on another.

- **Example 2**: A frequency distribution for the class level of students in Professor Mahmout's introductory statistics course is shown below.

Class	Frequency
Freshman	6
Sophomore	15
Junior	12
Senior	7
Total	**40**

Example 2: Tree diagram

- The tree diagram shows the probability for each combination of selecting two students without replacement.

Class	Frequency
Freshman	6
Sophomore	15
Junior	12
Senior	7
Total	**40**

1st Student	P(1st student)	2nd Student	P(2nd student)	Event	P(A & B)
Fr.	$^{6}/_{40}$	Fr.	$^{5}/_{39}$	Fr., then Fr.	$^{6}/_{40} \bullet {}^{5}/_{39} \approx .0192$
		So.	$^{15}/_{39}$	Fr., then So.	$^{6}/_{40} \bullet {}^{15}/_{39} \approx .0577$
		Jr.	$^{12}/_{39}$	Fr., then Jr.	$^{6}/_{40} \bullet {}^{12}/_{39} \approx .0462$
		Sr.	$^{7}/_{39}$	Fr., then Sr.	$^{6}/_{40} \bullet {}^{7}/_{39} \approx .0269$
So.	$^{15}/_{40}$	Fr.	$^{6}/_{39}$	So., then Fr.	$^{15}/_{40} \bullet {}^{6}/_{39} \approx .0577$
		So.	$^{14}/_{39}$	So., then So.	$^{15}/_{40} \bullet {}^{14}/_{39} \approx .1346$
		Jr.	$^{12}/_{39}$	So., then Jr.	$^{15}/_{40} \bullet {}^{12}/_{39} \approx .1154$
		Sr.	$^{7}/_{39}$	So., then Sr.	$^{15}/_{40} \bullet {}^{7}/_{39} \approx .0673$
Jr.	$^{12}/_{40}$	Fr.	$^{6}/_{39}$	Jr., then Fr.	$^{12}/_{40} \bullet {}^{6}/_{39} \approx .0462$
		So.	$^{15}/_{39}$	Jr., then So.	$^{12}/_{40} \bullet {}^{15}/_{39} \approx .1154$
		Jr.	$^{11}/_{39}$	Jr., then Jr.	$^{12}/_{40} \bullet {}^{11}/_{39} \approx .0846$
		Sr.	$^{7}/_{39}$	Jr., then Sr.	$^{12}/_{40} \bullet {}^{7}/_{39} \approx .0538$
Sr.	$^{7}/_{40}$	Fr.	$^{6}/_{39}$	Sr., then Fr.	$^{7}/_{40} \bullet {}^{6}/_{39} \approx .0269$
		So.	$^{15}/_{39}$	Sr., then So.	$^{7}/_{40} \bullet {}^{15}/_{39} \approx .0673$
		Jr.	$^{12}/_{39}$	Sr., then Jr.	$^{7}/_{40} \bullet {}^{12}/_{39} \approx .0538$
		Sr.	$^{6}/_{39}$	Sr., then Sr.	$^{7}/_{40} \bullet {}^{6}/_{39} \approx .0269$

Example 3

- The table below shows data about a group of asthma sufferers.
- If two people are selected at random, find:
  - The probability they are both heavy smokers.
  - The probability they are both women.

	Nonsmoker	Light Smoker	Heavy Smoker	Total
Men	327	40	45	412
Women	388	37	38	463
Total	715	77	83	875

Assessment

Pgs. 202-203:

#'s 4.109 - .119