P-Values

Section 9.5

Statistics, MATH 181

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The Concept of *P*-Values

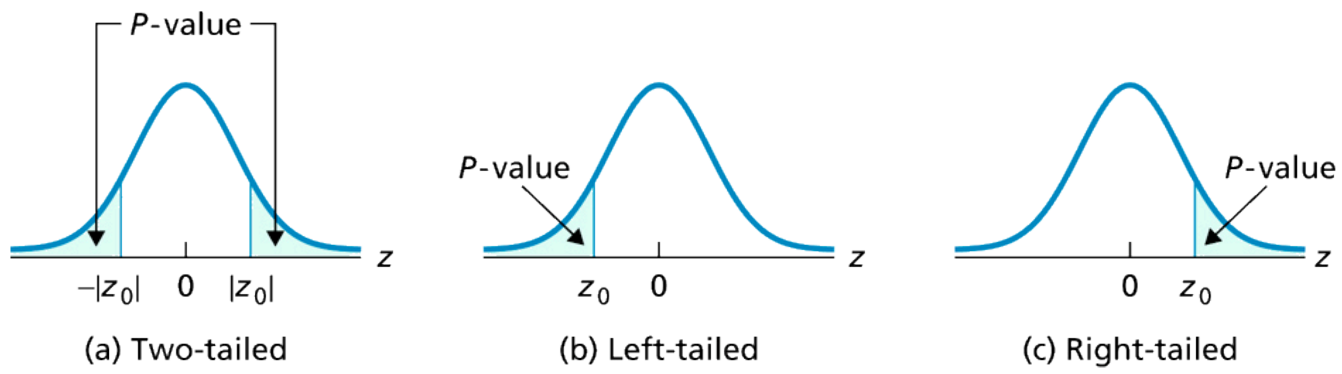
- From section 9.3, we devised a procedure to perform a hypothesis test with a ***known*** population standard deviation.
  - This procedure used the ***critical-value approach*** to hypothesis testing.
- In this section, we discuss another approach to hypothesis testing: the ***P-Value approach***.
  - Roughly speaking, the *P*-value tells how likely it is to *actually* observe the value obtained if the null hypothesis is actually true.

The *P*-Value Approach

- The ***P-Value*** is the percentage of samples that would yield a value of the test statistic as extreme as or more extreme than the observed value, if the null hypothesis is true.
  - A small *P*-value provides evidence ***against*** a null hypothesis; large *P*-values do not.
    - The smaller the *P*-value, the stronger the evidence is against the null hypothesis.

P-Values: Where They Come From

- Recall that the test statistic for a one-mean *z*-test is given by the formula: $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$
- Use the diagrams below to interpret the context of a *P*-value for each type of test:
  - Z0 is the observed value of the test statistic.
  - Remember, we interpret the area under the standard normal curve as a probability that an observation lies in that area.



EXAMPLE I

- From Exercise 9.63 on Pg. 428, we set out to determine if the mean length of imprisonment for car thieves in Sydney differed from the national mean in Australia. Using the sample mean of 17.8 and σ= 6.0, we arrived at a value for the test statistic of z = 1.83. Obtain and interpret the *P*-value of this hypothesis test.
  - Because we conducted a two-tailed test, the *P*-value is the probability of observing a value of z beyond 1.83, in either direction. That probability corresponds to the area under the standard normal curve outside of ±1.83, which is 1 - 0.9328 = **0.0672**.
  - The *P*-value for our hypothesis test would be 0.0672. This means that we would observe a value of z of 1.83 or greater only about 7 times in 100, or about 7% of the time.

The *P*-Value at the Significance Level

- The *P*-value of a hypothesis test can also be interpreted as the *observed significance level* for the hypothesis test.
  - More plainly put, it identifies the exact significance level at which the value of the test statistic *z* would lie ON the critical value for our significance interval.
- How do we decide to reject or not reject H_0?
  - If the *P*-value is less than or equal to the specified significance level, reject the null hypothesis, H_0.
    - If $P \leq \alpha$, then reject H_0.
  - Otherwise, do not reject the null hypothesis H_0.

Example 2: Exercise #9.118

- The mean retail price of all agriculture books is given as \$66.52, with a standard deviation of \$8.45.
- A sample of 28 books at this year's retail prices shows a sample mean of \$63.88.
  - At the 10% significance level, do the data provide evidence to conclude that this year's retail price has changed from the previous year's mean?

No Significance Level? No Time To Panic.

- In many cases, a significance level or critical value may not be given as part of a hypothesis test.
- Instead, we may simply obtain a *P*-value and use it to describe the strength against the null hypothesis, H_0.
  - The table at the right describes the criteria for strength against H_0.

P-value	Strength *against* H_0
$P > 0.10$	Weak or none
$0.05 < P \leq 0.10$	Moderate
$0.01 < P \leq 0.05$	Strong
$P \leq 0.01$	Very Strong

Assessment

Pgs. 446-447:

#'s 9.100 - 9.102, 9.118 - 9.122